



NEWTON'S FORWARD
&
NEWTON'S BACKWARD
INTERPOLATION

NEWTON'S FORWARD INTERPOLATION

- Formula of Newton's Forward Interpolation

$$y_n(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \dots$$
$$+ \frac{p(p-1)(p-2) \dots (p-n+1)}{n!}\Delta^n y_0$$

Here :- $p = \frac{x-x_0}{h}$

EXAMPLE

- Find The value Of $\tan 0.12$

x	0.10	0.15	0.20	0.25	0.30
$y = \tan x$	0.1003	0.1511	0.2027	0.2553	0.3093

SOLUTION

X	Y	Δ	Δ^2	Δ^3	Δ^4
0.10	0.1003				
		0.0508			
0.15	0.1511		0.0008		
		0.0516		0.0002	
0.20	0.2027		0.0010		0.0002
		0.0526		0.0004	
0.25	0.2553		0.0014		
		0.0540			
0.30	0.3093				

Applying Newton's Forward Difference Interpolation Formula.

$$y_n(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

Here $y_n(x) = \tan(0.12)$

$$\therefore p = \frac{x-x_0}{h} = \frac{0.12-0.10}{0.05} = \frac{0.02}{0.05} = 0.4$$

$$\therefore y_n(x) = 0.1003 + 0.4(0.0508) + \frac{0.4(0.4-1)}{2} 0.0008 + \frac{0.4(0.4-1)(0.4-2)}{6} 0.0002 + \frac{0.4(0.4-1)(0.4-2)(0.4-3)}{24} 0.0002$$

$$y_n(x) = 0.1205$$

NEWTON'S BACKWARD INTERPOLATION

- Formula of Newton's Backward Interpolation

$$y_n(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \dots + \frac{p(p-1)\dots(p+n-1)}{n!}\nabla^n y_n$$

$$\text{Here :- } p = \frac{x-x_n}{h}$$

EXAMPLE

Consider Following Tabular Values Determine $y(300)$

x	50	100	150	200	250
y	618	724	805	906	1032

SOLUTION

X	Y	∇	∇^2	∇^3	∇^4
50	618				
		106			
100	724		-25		
		81		45	
150	805		20		-40
		101		5	
200	906		5		
		126			
250	1032				

Applying Newton's Backward Difference Interpolation Formula.

$$y_n(x) = y_0 + p\nabla y_n + \frac{p(p-1)}{2!}\nabla^2 y_n + \frac{p(p-1)(p-2)}{3!}\nabla^3 y_n + \frac{p(p-1)(p-2)(p-3)}{4!}\nabla^4 y_n$$

Here:- $y_n(x) = y_n(300)$

$$\therefore p = \frac{x-x_n}{h} = \frac{300-250}{50} = 1$$

$$\begin{aligned}\therefore y_n(x) &= 1032 + 126 + \frac{1(1+1)}{2!}25 + \frac{1(1+1)(1+2)}{3!}5 + \frac{1(1+1)(1+2)(1+3)}{4!}(-40) \\ &= 1032 + 126 + 25 + 5 - 4\end{aligned}$$

$$y_n(300) = 1148$$



THANK YOU!